

**Paper Body:** We have been developing ultrasound (US) high spatial resolution echo imaging and high accuracy tissue displacement vector measurement methods. For instance, simultaneous use of our previously developed multidimensional autocorrelation method (MAM) and a lateral modulation (LM) using steered beams or steered plane waves with a high frame rate permits a high accuracy measurement. In this report, we generate over-determined sets of data from large numbers of LMs for effectively performing the least squares solution (LST), averaging of measurements (AVE), and coherent superposition (compounding) of beams or waves (CS). Previously, data with a realistic echo SNR was used, but only a single LM was performed. Nevertheless, this generated the highest measurement accuracy with MAM to date. For spherical transmission and dynamic reception synthetic focusing (SF) scanning, over-determined datasets with high LM and independent frequencies yield the highest strain measurement accuracies for LST, AVE and CS (SDs 0.16 or 0.20% for 16 or 10 steered beams), whereas for plane wave (PW) transmissions, accuracies of 0.19 to 0.21% are obtained using 14 or 12 steered waves respectively. The accuracy of CS is the lowest of all the methods, particularly for SF scanning. Weighted over-determined systems (LST) using the ZZLBs about the displacements in respective generated beam directions as weights are also used. In addition, the super-resolution echo imaging preformed on only 10 waves CS achieves almost the same spatial resolution as that of a single LM with SF scanning.

**Background:** We have been developing ultrasound (US) high spatial resolution echo imaging and high accuracy tissue displacement vector measurement methods. For instance, simultaneous use of our previously developed multidimensional autocorrelation method (MAM) and a lateral modulation (LM) using steered beams or steered plane waves with a high frame rate permits a high accuracy measurement. We also generate over-determined sets of data from large numbers of LMs for effectively performing the least squares solution (LST), averaging of measurements (AVE), and coherent superposition (compounding) of beams or waves (CS). Although small inclusions were visualized in [1], the synthetic aperture echo data used had low echo SNR due to the scanner employed. In [2], data with a more realistic echo SNR was used, but only a single LM was performed. Nevertheless, this generated the highest measurement accuracy with MAM to date. We should clarify accuracies achievable via LST, AVE and CS on high SNR echo data.

**Aims:** To clarify the tissue displacement and strain vector measurement accuracies achievable using the above methods for realistic SNR echo data obtained from an agar phantom [2] (US freq., $f = 7.5$ MHz).

**Methods:** The agar phantom had a cylindrical inclusion (10mm dia.) with higher shear modulus than the surrounding, and was compressed in a lateral direction (about -2%). The MAM was used. Super-resolution imaging was also performed by CS of plane waves.

**Results:** Figure 1 shows, for scanning with parabolic transmission and apodized reception, and spherical transmission and dynamic reception synthetic focusing (SF), the standard deviations (SDs) of measured axial, lateral and shear strains vs LM frequency. Note that although increasing LM frequency decreases the axial frequency, increments up to $3/8f$ increase the measurement accuracy of axial strain together with that of lateral strain. This is achieved by the simultaneous measurements of both axial and lateral displacements.
Figure 2 shows, for SF scanning, lateral strain images obtained using single or plural LMs with the indicated LM frequencies. In [2], the most accurate lateral strain measurement achieved with a single LM was with an LM of 1/2f and had a SD of 0.28%. Here, as shown in the figure, using an LM frequency = f yields a more accurate measurement, with SD of 0.22%. Furthermore, by using plural high LM frequency beams, smaller SDs of 0.16%, 0.16% and 0.20% were respectively achieved for: 16 beams LST (1/8f to f with an interval of 1/8f), 10 beams AVE and 10 beams CS (1/2f to f with an interval 1/8f) (images omitted). However, using independent LM frequencies obtained using large steering angle differences (Fig. 2) achieves the same accuracies using fewer beams, i.e., 8 beams LST using (1, 1/2, 3/4, 1/4) f, and 6 beams AVE using (1, 1/2, 3/4) f and 4 beams CS using (1, 1/2) f.

When using plane wave (PW) transmissions with spherical dynamic reception focusing and Gaussian reception apodization, single LMs with frequencies f/2 and f yielded SDs of 0.50% [2] and 0.42% respectively (images omitted). Similar combinations for LST, AVE and CS yielded respectively: 0.27% (6 waves, 2 each with LMs of f, 3/4f and f/2), 0.27% (the same 6 waves) and 0.31% (4 waves, 2 each with LMs of f and 3/4f). When using more waves, however, only 14, 12 and 14 waves, for LST, AVE and CS respectively, yielded more accurate measurements than the best single LM with SF scanning (0.22 %), i.e., SDs of 0.19, 0.20 and 0.21%, respectively.

Weighted over-determined systems (LST and AVE) using variances of displacements were also be used, and preliminary results obtained as follows. Specifically, in this report, the variances were estimated using...
the Ziv-Zakai lower bound (ZZLB) at respective positions [3], although the estimation was also be able to be performed in a locally stationary process [4]. Because the confidence of respective MAM equations about displacement vector components can be considered as those of equations about displacements in the same directions as those of the correspondingly generated beams [5], ZZLBs calculated using frequencies and bandwidths in the respective beam directions can be used as weights for the corresponding MAM equations. That is, the multidimensional Doppler equations, respective of which has a beam direction \( i \) (\( i = 1 \) to \( 2 \)) is weighted as follows,

\[
W_i(\theta_x + \frac{\partial}{\partial x} \theta_x + \frac{\partial}{\partial y} \theta_y) = 0,
\]

where the weight \( W_i \) (\( i = 1 \) or \( 2 \)) is expressed by

\[
W_i = 1/\sqrt{\text{ZZLB}_i}
\]

about a displacement \( u_i \) in the beam direction \( i \). Alternatively, when performing AVE, with respect to the calculated results \((u_{xi}, u_{yi})\) using a beam \( i \) (\( i = 1 \) to \( 2 \)), weighted mean components are obtained as

\[
(u_x, u_y) = \frac{\sum W_i (u_{xi}, u_{yi})}{\sum W_i} \quad \text{or} \quad (u_x, u_y) = \left(\frac{\sum W_i u_{xi}}{\sum W_i}, \frac{\sum W_i u_{yi}}{\sum W_i}\right)
\]

using estimates of respective displacement components \((u_{xi}, u_{yi})\) [omitted here].

In this report, being different from the use for a spatially variant regularization in [3], spatially uniform echo and correlation SNRs are assumed. The frequencies and bandwidths were estimated using instantaneous frequencies or 1st moment, and 2nd moment, respectively. For instance, for SF scanning with 8 beams with LM frequencies \((1, 1/2, 3/4, 1/4) \times f\) and for PW transmissions with 6 waves with \((1, 1/2, 3/4) \times f\), the measurement accuracies were not so improved (SDs, 0.16% and 0.25%).

For super-resolution imaging using SF scanning and a single LM frequency of \( 1/2f \), superposition using CS with 10 plane waves \((1/4f, 3/8f, 1/2f, 5/8f, 3/4f)\) yielded a high spatial resolution (Figure 3), particularly demonstrating strong scattering depicted by the circle indicated. For comparison, our previously reported super-resolution image using CS with two plane waves (i.e., LM with \( 1/2 \times f \)) is also shown [6]. Corresponding spectra are also shown.

Conclusions: For SF scanning, over-determined datasets with high LM and independent frequencies yielded the highest strain measurement accuracies for LST, AVE and CS (SDs 0.16 or 0.20% for 8 or 6 steered beams), respectively, whereas for PW transmissions, accuracies of 0.19 to 0.21% were obtained using 14 or 12 steered waves, respectively. The accuracy of CS was the lowest of all the methods, particularly for SF scanning, respectively. For an over-determined system (LST and AVE), independent high lateral frequency spectra (even if the measurement accuracy is low solo; this is a low raw echo SNR case, for instance, see [1]) rather than dependent spectra with small steering angle differences should be used; for CS, particularly for a low echo SNR case (e.g., see [1]), use of dependent spectra is effective. For PW transmissions, i.e., lower echo SNRs than those of corresponding SF cases, as previously reported by us (e.g., [6] and [7] cases \( \leq 1/2f \); and [1] case with a low raw echo SNR), AVE yields accurate results. Weighted over-determined systems (LST) using the ZZLBs about the displacements in respective generated beam directions as weights were also used. Only the use of frequencies and bandwidths for estimating ZZLBs did not improve the measurement accuracy of a displacement vector so much. Then, echo and correlation SNRs should also be used in the near future. Weighted averaging results will also be specifically reported. Stationary estimate of variance will also be used. Simultaneous spatially variant displacement-component-dependent regularization [4] will also be performed using variance predictions. In addition, the super-resolution echo imaging performed on only 10 waves CS achieved almost the same spatial resolution as that of a single LM with SF scanning.